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Supersymmetry in the false vacuum.

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Abstract

A metastable state, or a false vacuum, is not an eigenstate of the Hamiltonian in quantum field theory. Its energy density has a non-zero imaginary part equal to its decay width. Therefore, supersymmetry cannot be exact in the false vacuum. We calculate the size of this effect using the path integral approach.

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The Universe is believed to have arrived at its present state via a series of phase transitions, each of which involved a decay of some metastable quantum state. A good understanding of these transitions is essential for explaining the physical world at present. In particular, the baryon asymmetry of the Universe, dark matter, and many other cosmological and experimental data may owe their existence to the physics of the early Universe. Although the experimental evidence for supersymmetry has yet to be discovered, theories with low-energy supersymmetry provide an attractive explanation for the hierarchy of scales and are widely considered viable. It is of interest, therefore, to understand the effects of supersymmetry on the decay of a metastable state.

It is well-known that supersymmetry is broken at non-zero temperature [1]. There is, however, an additional effect, a generic feature of the false vacuum which is due to its instability *per se*, which can contribute to SUSY breaking¹ and be equally or more important than the finite-temperature effects.

Unlike the true vacuum, $|0\rangle$, a false vacuum $|0'\rangle$ is not an eigenstate of the Hamiltonian H [2]. Its decay width per unit volume, (Γ/V) , is equal to the imaginary part of the energy density, $Im\langle 0'|H|0'\rangle$, which must be non-zero if the false vacuum is to decay. On the other hand, by the usual argument, exact supersymmetry cannot be realized unless the vacuum energy is zero. The latter requirement implies that both the real and imaginary parts of the energy density must vanish. Indeed, the anticommutation relation between the supersymmetry generators Q_α

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\sigma_{\alpha\beta}^\mu P_\mu, \quad (1)$$

together with the statement of unbroken supersymmetry,

$$Q_\alpha|0'\rangle = Q_\beta^\dagger|0'\rangle = 0, \quad (2)$$

¹In application to the false vacuum, the term *SUSY breaking* is somewhat a misnomer because supersymmetry is not really broken. The unstable state that is considered to be a “vacuum” is simply not a part of the spectrum and the corresponding Fock space is ill-defined. Nevertheless, for all intents and purposes SUSY will appear broken to an observer in the false vacuum, which justifies the use of the term.

yield immediately that

$$\langle 0'|P_\mu|0'\rangle = 0 \implies \text{Im}\langle 0'|P_\mu|0'\rangle = 0 \implies \Gamma/V = 0, \quad (3)$$

in contradiction with the assumption that $|0'\rangle$ is a metastable state and thus has a non-zero decay width.

This simple line of reasoning shows that supersymmetry cannot be exact in any metastable state and that the vacuum in which SUSY is unbroken must be stable. The latter can serve as an independent argument for the stability of the supergravity vacuum with vanishing cosmological constant even in the presence of lower lying minima in the potential, a fact established in Ref. [3] by other means.

It is clear that the size of the SUSY breaking effects in the false vacuum is of order Γ/V . Below we will show that this is, in fact, the case and will compute the mass splitting $\Delta m^2 = m_B^2 - m_F^2 \sim \langle F \rangle$ for the bosonic and fermionic components of a chiral superfield which arises due to non-perturbative effects related to false vacuum instability. In general, there are other contributions to Δm^2 which can be computed in perturbation theory. For example, if the scalar potential is of the form Fig. 1, and if supersymmetry is unbroken in the true minimum, then there will be a contribution $(\Delta m^2)_R$ to Δm^2 from the real part of the energy density due to spontaneous or explicit (soft) SUSY breaking at $\phi = 0$. In realistic models one can assume $(\Delta m^2)_R \sim M_{SUSY}^2$. At finite temperature supersymmetry is always broken [1] which gives rise to a temperature-dependent contribution $(\Delta m^2)_T$. So, in general,

$$\langle F \rangle \sim \Delta m^2 = (\Delta m^2)_R + (\Delta m^2)_T + (\Delta m^2)_I \quad (4)$$

where $(\Delta m^2)_I$ is the non-perturbative piece induced by the false vacuum instability. The relative magnitudes of the terms in (4) are model-dependent. However, one can always construct a model in which the barrier separating the false vacuum from the true is sufficiently small, so that the last term in equation (4) is arbitrarily large, while the first two terms are much smaller.

We see that the supersymmetry breaking due to the instability of the false vacuum will dominate over that due to finite-temperature and other effects for a metastable state that is sufficiently short-lived, or, in other words, if $\Gamma/V \gg M_{SUSY}^4$ and $\Gamma/V \gg T^4$. In what follows we will neglect the $(\Delta m^2)_R$ and $(\Delta m^2)_T$ which can be calculated perturbatively in each given model and will concentrate on $(\Delta m^2)_I$.

We now evaluate the mass splitting of the fermion ψ and boson ϕ components in a model involving a single chiral superfield $\Phi = \{\phi, \psi, F\}$ with the scalar potential that has two non-degenerate local minima (Fig. 1).

In the path integral formulation a vacuum expectation value of some function $f(\phi)$ of the dynamical field can be written as

$$\langle f(\phi) \rangle = \lim_{J \rightarrow 0} \frac{\delta}{\delta J(x_0)} \ln \int [d\phi] \exp(-\frac{1}{\hbar} S[\vec{\phi}] + \int J f(\phi)), \quad (5)$$

where $S[\vec{\phi}]$ is the Euclidean action and $J(x)$ is the source.

The non-perturbative contribution to $f(\phi)$ can be evaluated in the semiclassical limit $\hbar \rightarrow 0$ using the saddle point approximation (see, *e.g.*, [2, 4]). The idea of the method is that one can expand the path integral around the finite-action solutions which are saddle points of the Euclidean action and are, therefore, determined by the variational equation

$$\delta S = 0. \quad (6)$$

For a fermion $\psi(x)$, the Euler-Lagrange equation (Euclidean equation of motion) corresponding to equation (6) is a first-order differential equation. The boundary conditions $\psi(x \rightarrow \infty) = 0$ are necessary to ensure the finiteness of the action. The only solution $\psi(x)$ of equation (6) satisfying these boundary conditions is the trivial one $\psi(x) \equiv 0$.

In contrast to fermions, the bosonic equation of motion corresponding to (6) is second-order and it allows for a non-trivial solution of finite action, the so called “bounce” [2]. The bounce dominates the path integral contribution to the imaginary part of the energy density, which was calculated in Refs. [2, 4, 6]. More specifically, the bounce for a scalar

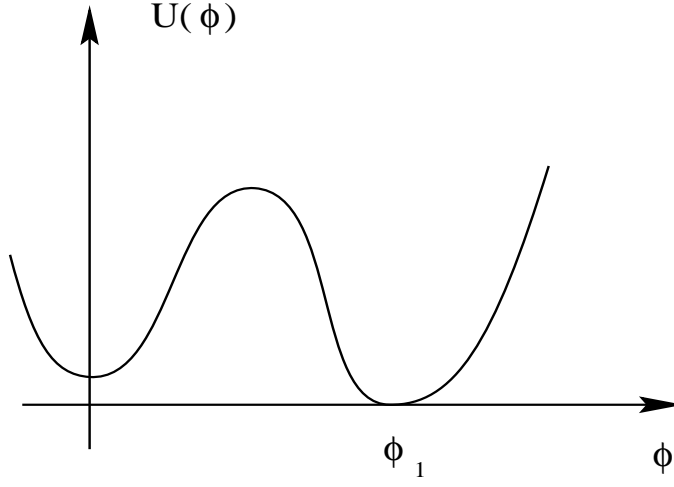


Figure 1: The false and the true vacua.

potential $U(\phi)$ (Fig. 1) is a nontrivial $O(4)$ -symmetric [5] field configuration determined by the equation²

$$\Delta \bar{\phi}(r) = \frac{\partial}{\partial \bar{\phi}} U(\bar{\phi}) \quad (7)$$

with the following boundary conditions:

$$\begin{cases} (d/dr)\bar{\phi}(r)|_{r=0} = 0 \\ \bar{\phi}(\infty) = 0 \end{cases} \quad (8)$$

where $r = \sqrt{x^2}$.

The potential $U(\phi)$ can be written as

$$U(\phi) = \frac{m_0}{2}\phi^2 + \sum_{n>2} c_n \phi^n. \quad (9)$$

and the boson mass, by definition, is

$$m_\phi^2 = \left\langle \left(\frac{\partial^2 U(\phi)}{\partial \phi^2} \right) \right\rangle$$

² If the potential depends on several scalar fields, then finding the bounce numerically becomes a non-trivial task. In this case, the method of Ref. [7] is useful.

$$\begin{aligned}
&= \frac{\int \left(\frac{\partial^2 U(\phi)}{\partial \phi^2} \right) e^{-S[\phi]} [d\phi]}{\int e^{-S[\phi]} [d\phi]} \\
&= m_0^2 + \frac{\int f(\phi) e^{-S[\phi]} [d\phi]}{\int e^{-S[\phi]} [d\phi]}
\end{aligned} \tag{10}$$

where $f(\phi) = (\partial^2/\partial\phi^2) \sum_{n>2} c_n \phi^n$. We note that $f(0) = 0$.

We now apply the saddle point method to calculate the quantity $\Delta m^2 = m_\phi^2 - m_0^2$. First we will compute the normalization factor

$$N = \int e^{-S[\phi]} [d\phi] \tag{11}$$

in the denominator of the second term in (10). This part of the calculation will follow closely that of Refs. [2, 4]. For simplicity, we consider the case of one space-time dimension t ; generalization to $d = 4$ is straightforward and will follow.

The bounce of equation (7) is centered at $t = 0$ because $\bar{\phi}'(0) = 0$. Let's consider a contribution from a single bounce $\phi(t - t_j)$ centered at $t = t_j$, $-T/2 < t_j < T/2$, where T is some large quantity taken eventually to infinity. A classical field configuration corresponding to n widely separated bounces will also be a saddle point of the integrand with the Euclidean action $nS[\bar{\phi}]$. Therefore, we have to sum the contributions of all such saddle points and integrate over the positions of the bounces.

We are interested in the limit $T \rightarrow \infty$. The n bounces centered at t_1, t_2, \dots, t_n , where $-T/2 < t_n < \dots < t_2 < t_1 < T/2$, are separated by vast regions where the field is nearly zero. Therefore the determinant part of the path integral can be written as a product

$$\sqrt{(\omega/\pi\hbar)} e^{-\omega T/2} K^n \tag{12}$$

where $\omega = U''(0)$ and K is defined so as to yield the correct answer for $n = 1$.

We must now integrate over the locations of the centers of the bounces.

$$\int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{t_1} dt_2 \dots \int_{-T/2}^{t_{n-1}} dt_n = \frac{T^n}{n!} \quad (13)$$

The summation of contributions with different n gives:

$$\begin{aligned} \sum_{n=1}^{\infty} \left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2} K^n (e^{-S[\bar{\phi}]/\hbar})^n \frac{T^n}{n!} = \\ \left(\frac{\omega}{\pi\hbar}\right)^{1/2} \exp(-\omega T/2 + K e^{-S[\bar{\phi}]/\hbar} T) \end{aligned} \quad (14)$$

This is the same as the corresponding result in [4]. The quantity K is the correction to the energy which determines the transition probability. We will come back to it later.

Now we treat the numerator of the second term in (10). This is basically the same, except now the configuration which comprises n widely separated bounces, $\phi = \sum_{j=1}^n \bar{\phi}(t_0 - t_j)$, will contribute

$$\sum_{j=1}^n f[\bar{\phi}(t_0 - t_j)] e^{-nS[\bar{\phi}]/\hbar}, \quad (15)$$

where the property $f(0) = 0$ is essential.

Like before, we have to integrate over the positions of the bounces:

$$\int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{t_1} dt_2 \dots \int_{-T/2}^{t_{n-1}} dt_n \sum_{j=1}^n f[\bar{\phi}(t_0 - t_j)] \quad (16)$$

To evaluate the integral, we exchange the order of summation and integration. Then we let each variable t_i run over the entire interval $[-T/2, T/2]$. The latter change would lead to a double counting of the contributions differing by permutations of the identical bounces. We therefore divide by $(n!)$, the number of permutations of n bounces:

$$\sum_{j=1}^n \frac{1}{n!} \int_{-T/2}^{T/2} dt_1 \int_{-T/2}^{T/2} dt_2 \dots \int_{-T/2}^{T/2} dt_n \bar{\phi}(t_0 - t_j) =$$

$$\frac{1}{n!} \sum_{j=1}^n \int_{-T/2}^{T/2} dt_1 \dots \left\{ \int_{-T/2}^{T/2} dt_j f[\bar{\phi}(t_0 - t_j)] \right\} \dots \int_{-T/2}^{T/2} dt_n \quad (17)$$

In the last expression, the integration over t_j gives (in the limit of large T) $\int_{-\infty}^{\infty} \bar{\phi}(\xi) d\xi$, while each of the other integrals gives a factor of T . The sum over j has n identical terms and is equal to n times the value of each. The factor n then cancels with that of $n!$. Altogether we get

$$\left\{ \int_{-\infty}^{\infty} f[\bar{\phi}(\xi)] d\xi \right\} \frac{T^{n-1}}{(n-1)!} \quad (18)$$

We notice in passing that the expression (18) differs from (13) in two respects: (1) there is a constant factor of $\int f[\bar{\phi}(\xi)] d\xi$; (2) it has one less power of T .

And, finally, we sum over n : The $n = 0$ component will not contribute; therefore, the sum starts from the $n = 1$ term:

$$\begin{aligned} & \sum_{n=1}^{\infty} \left(\frac{\omega}{\pi\hbar} \right)^{1/2} \left\{ \int_{-\infty}^{\infty} f[\bar{\phi}(\xi)] d\xi \right\} e^{-\omega T/2} K^n (e^{-S[\bar{\phi}]/\hbar})^n \frac{T^{n-1}}{(n-1)!} = \\ & \left\{ \int_{-\infty}^{\infty} f[\bar{\phi}(\xi)] d\xi \right\} K e^{-S[\bar{\phi}]/\hbar} \left(\frac{\omega}{\pi\hbar} \right)^{1/2} \sum_{n=1}^{\infty} e^{-\omega T/2} K^{n-1} (e^{-S[\bar{\phi}]/\hbar})^{n-1} \frac{T^{n-1}}{(n-1)!} = \\ & \left\{ \int_{-\infty}^{\infty} f[\bar{\phi}(\xi)] d\xi \right\} K e^{-S[\bar{\phi}]/\hbar} \left(\frac{\omega}{\pi\hbar} \right)^{1/2} \exp(-\omega T/2 + K e^{-S[\bar{\phi}]/\hbar} T) \end{aligned} \quad (19)$$

As we now divide expression (19) by (14), we obtain a simple result:

$$m_{\phi}^2 = m_0^2 + \Delta m^2 = m_0^2 + \left\{ \int_{-\infty}^{\infty} f[\bar{\phi}(\xi)] d\xi \right\} K e^{-S[\bar{\phi}]/\hbar} \quad (20)$$

where the factor K is that computed in [4]³:

³ In applications, one is often interested in transitions between phases with different symmetries. In this case, some additional zero modes appear in the determinant of equation (21) due to the Goldstone phenomenon. The corresponding N zero eigenvalues should then be omitted from the “primed” determinant in (21) and an additional factor $[\int \bar{\eta}^2(x) d^4x]^{N/2}$ will appear in the expression for Δm^2 [6].

$$|\Delta m^2| = \left\{ \int_{-\infty}^{\infty} f[\bar{\phi}(\xi)] d\xi \right\} \left(\frac{S[\bar{\phi}]}{2\pi\hbar} \right)^2 e^{-S[\bar{\phi}]/\hbar} \left| \frac{\det'[-\partial_\mu^2 + U''(\bar{\phi})]}{\det[-\partial_\mu^2 + U''(0)]} \right|^{-1/2} \times (1 + O(\hbar)) \quad (21)$$

where \det' stands for the determinant from which all zero eigenvalues are omitted.

As was explained earlier, there is no analog of the bounce for fermions, so that the correction (21) applies to bosons only.

The above discussion dealt with the zero-temperature tunneling. It is straightforward to generalize the expression (21) to the finite-temperature case [8]. The changes will amount to replacing the Euclidean action of the bounce with the action of a three-dimensional bounce S_3 divided by the temperature, and the factor $(S/\hbar)^2$ with $(S_3/\hbar)^{3/2}/T^{1/2}$.

In summary, we have computed the mass splitting between the components of a chiral superfield which is related to the instability of the false vacuum. If the metastable state decays sufficiently fast, this effect may dominate over the SUSY breaking finite-temperature effects, as well as the soft SUSY breaking terms of phenomenologically acceptable size. We have also given an independent argument for the supersymmetric vacuum stability in supergravity.

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